RAMAKRISHNA MISSION VIDYAMANDIRA

(Residential Autonomous College affiliated to University of Calcutta)

B.A./B.Sc. SIXTH SEMESTER TAKE-HOME TEST/ASSIGNMENT, JULY 2020

THIRD YEAR [BATCH 2017-20]

Date : 03/07/2020 **MATHEMATICS (Honours)**

Time : 11am – 5pm **Paper : VII & VIII** Full Marks : 85

Instructions to the Candidates

- Write your Name, College Roll no, Subject and Paper Number on the top of the Answer Script and on the text body of the mail.
- Read the instructions given at the beginning of each paper/group/unit carefully.
- Groups A - F are to be answered by everyone. Along with these, each candidate must answer any one of the two Groups G and H.
- Only handwritten (by blue/black pen) answer-scripts will be admissible.
- Each paper/group must be answered in a single booklet.
- All the pages of your answer scripts must be numbered serially by hand.
- In the last page of your answer-scripts, please mention the total number of pages written so that we can verify it with that of the scanned copy of the scripts sent by you.
- For an easy scanning of the answer scripts and also for getting better image, students are advised to write the answers in single side and they must give a minimum 1 inch margin at the left side of each paper.
- After the completion of the test, you should scan the entire answer scripts by using Clear Scan : Indy Mobile App /other Scanner devices and make a file in your own name and send or share them as a PDF file to [redacted]

Paper VII

Group - A (Analysis IIIB)

Answer all the questions from this group. [3 x 5 = 15 marks]

1. Show that the following integral converges for $m > 0$, $n > -1$. [5]

$$\int_0^1 x^{m-1}(1-x)^{n-1}\log x dx$$

2. (a) Examine the convergence of [2]

$$\int_0^2 \frac{\log x}{\sqrt{2-x}}dx.$$

(b) Show that

$$\iiint (x+y+z+1)^2 dxdydz = \frac{31}{60},$$

taken throughout the region within the tetrahedron bounded by the planes $x=0$; $y=0$; $z=0$, $x+y+z=1$. [3]

3. Verify Gauss' theorem for the surface integral

$$\iint_S (2xy+z)dydz + y^2dzdx - (x+3y)dxdy,$$

taken over the region bounded by $2x+2y+z=6$; $x=0$; $y=0$; $z=0$. [5]

Group - B (Number Theory)

Answer any 2 questions from this group. [2 x 5 = 10 marks]

4. (a) Use mathematical induction to prove that [3]

$$24|(2\cdot 7^n + 3\cdot 5^n - 5), \forall n \geq 1.$$

(b) For any integer a, prove that $3|a(2a^2+7)$. [2]

5. (a) Without using mathematical induction prove that $27|(2^{5n+1}+5^{n+2}), \forall n \geq 1$. [2]

(b) If a is odd, not divisible by 3, prove that $a^2 \equiv 1 (\bmod 24)$. [3]

6. Find the smallest positive integer n, having exactly 18 positive divisors. [5]

Group - C (Probability Theory)

Answer all questions from this group. [3 x 5 = 15 marks]

7. There are (2n+1) coupons bearing numbers $1, 2, 3, ..., (2n+1)$ in a box. If three coupons are drawn at random from the box, then find the probability that the chosen number are in A.P. [5]

8. Find the value of the constant k so that the function given by

$$f(x) = \begin{cases} kx(2-x), & 0 < x < 2, \\ 0, & \text{otherwise} \end{cases}$$

is a probability density function. Construct the distribution function and compute $P(X > 1)$. [5]

9. Let X follows normal distribution with mean μ and standard deviation σ, then prove that

$$\mu_{2k+1} = \sigma^2\mu_{2k} + \sigma^3\frac{d\mu_{2k}}{d\sigma},$$

where μ_k is the k-th central moment. [5]

Group - D (Complex Analysis)

Answer all the questions from this group. [2 x 5 = 10 marks]

10. (a) Show that, z and z' corresponds to diametrically opposite points on the Riemann sphere if $z\bar{z}' = -1$.
(Use the equation of Riemann sphere as: $x^2 + y^2 + z(z-1) = 0$) [3]

(b) Show that $\overline{e^{iz}} = e^{i\bar{z}}$ iff $z = n\pi$, for $n \in \mathbb{Z}$. [2]

11. (a) Let f and g be two differentiable functions in a domain D and Re f(z)= Re g(z) on D. Then show that f(z)= g(z)+ constant, $\forall\ z \in$ D. [3]

(b) If f(z) is real valued and differentiable on a domain D, then show that f(z) is constant on D. [2]

Paper VIII

Group - E (Analytical Statics)

Answer any two questions from this group. [2 x 7.5 = 15 marks]

12. Three forces P, Q and R act along the sides of the triangle formed by the lines $x + y = 1$; $x - y = -1$ and $y = 3$. Find the magnitude of their resultant and the equation of the line of action of their resultant. [7.5]

13. A smooth paraboloid of revolution is fixed with its axis vertical and vertex upwards; on it is placed a heavy elastic string of unstreched length $2\pi c$; when the string is in equilibrium show that it rests in the form of a circle of radius

$$\frac{ac\pi\lambda}{a\pi\lambda - cW}$$

where W is the weight of the string, λ is its modulus of elasticity and a is the latus rectum of the generating parabola. [7.5]

14. A square board is hung flat against a wall by means of a string fastended to the two extremities of the upper edge and hung round a perfectly smooth nail. When the length of the string is less than the diagonal of the board, show that there are three positions of equilibrium. Show that the equilibrium in the position of symmetry is unstable. [7.5]

Group - F (Computer fundamentals and Programming in C)

Answer any 2 questions from this group. [2 x 5 = 10 marks]

15. (a) Let $\mathcal{B}$ be a Boolean algebra and $X, Y, Z \in \mathcal{B}$. Show, that

$$(X + Y + Z) \cdot (X + Y + Z') \cdot (X + Y' + Z) \cdot (X' + Y + Z) = X \cdot Y + Y \cdot Z + Z \cdot X$$

(b) Consider a company committee of 4 people A, B, C and D. The Director D has a veto power, i.e. he can reject a decision even if every one else agrees with it. But, to accept a proposal, there has to be a clear majority among the members. Considering a proposal is presented in front of the committee, find a Boolean function that will give output as 1 if the proposal is passed.

(c) Prove or disprove the following statement with proper justification
"A Boolean function f of 3 independent Boolean variables has a conjunctive normal form iff it has a disjunctive normal form." [2+2+1]

16. Consider an airport luggage handling automated system that will accept a luggage and measure its dimensions. There are 3 internal sensors A, B and C, that will glow according as the length, breadth and height of the luggage are within the maximum allowed values L, B and H respectively. The luggage will be allowed as a cabin luggage if all the dimensions are within range. Otherwise, it will be considered as a main luggage, provided its weight is not more than W; failing which the luggage will be rejected. If a fourth sensor D glows when the luggage is overweight, draw 3 circuit diagrams that will glow if the luggage is

    (a) a cabin luggage,

    (b) a main luggage,

    (c) overweight. [2+2+1]

17. (a) What are the basic differences and similarities between a compiler and an interpreter?

    (b) What is an assembly level language and how is it different from a machine level and a high level language?

    (c) What will be the output, after the following C-code is executed?

```
#include<stdio.h>
void main()
{ int x = 5;
for( x = -1 ; x < 3 ; ++x )
printf("x = %d;", x);
printf("\n The final value of x = %d.", x);
}
```

[1+1+3]

Optional Portion - Answer any one of Group G and H

Group - G (Tensor Calculus)

Answer any 2 questions from this group. [2 x 5 = 10 marks]

18. Prove that (where all the symbols have their usual significance): [5]

$$\frac{\partial^2 \bar{x}^a}{\partial x^b \partial x^c} = -\frac{\partial \bar{x}^a}{\partial x^u}\frac{\partial \bar{x}^h}{\partial x^b}\frac{\partial \bar{x}^w}{\partial x^c}\frac{\partial^2 x^u}{\partial \bar{x}^h \partial \bar{x}^w}.$$

19. If T_{ijkl} is a tensor which satisfies the relations $T_{ijkl} + T_{ijlk} = 0$, $T_{ijkl} + T_{jikl} = 0$ and $T_{ijkl} + T_{iklj} + T_{iljk} = 0$ and a_{lm} is a non-zero tensor such that $T_{hijk}a_{lm} + T_{lmhi}a_{jk} + T_{jklm}a_{hi} = 0$. Prove that $T_{ijkl} = 0$. [5]

20. If $P(i,j,k) = \left\{ \begin{matrix} i \\ j\ k \end{matrix} \right\} + \delta^i_j A_k + \delta^i_k A_j$, where A_l is a covariant vector and B^{ij} is any skew-symmetric tensor, show that $B^{sq}P(l,q,s) = 0$. [5]

Group - H (Topology)

Answer any 2 questions from this group. [2 x 5 = 10 marks]

21. For each pair of positive integers a and b, let $U(a,b) = \{na + b : n \in \mathbb{Z}\} \cap \mathbb{N}$. Prove that $\{U(a,b) : \gcd(a,b) = 1\}$ is a basis for some topology on $\mathbb{N}$. [5]

22. Show that a countable first countable space is 2nd countable. [5]

23. Let $p : \mathbb{R} \to \mathbb{R}$ be a polynomial. Show that if K is closed in $\mathbb{R}$, then $p(K)$ is also closed in $\mathbb{R}$. [5]

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